A/B Testing: Next Steps

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Learning Goals

- Explain how CUPED decreases variance of estimates in an A/B Test
- Implement a CUPED analysis in R
- Explain why and when one needs to adjust standard errors in A/B Test analysis using linear regression
- Implement standard error adjustments in R
- Define the SUTVA assumption and analyze whether the it is appropriate in a particular setting
- Explain alternative experiment designs that allow unbiased treatment effect estimation when SUTVA would be violated in a standard test design

Where Are We Now?

So far:

- Randomization as a modus operandi to overcome selection effects and omitted variable bias
- Design and analysis of “standard” A/B tests

This lecture: Tweaking the standard design

- Reducing the variance of our estimates
- Correct inference when treatment allocation is at a coarser level than the data we analyse
- How to handle violations of a *hidden* assumption

1/ Variance Reduction with CUPED

What is CUPED?

CUPED: Controlled-Experiment using Pre-Experiment Data

- A technique to increase the power of randomized controlled trials in A/B tests.

How does it work?

Let's start with some data...

Testing the Effectivness of a New Recommender

Business questions: Does the new recommender system increase spending?

Test setting: Online Website, recommender system

Unit: A consumer

Treatments: control group, new recommender system

Reponse: spending in the next 14 days

Selection: all consumers who purchased in last 60 days

Assignment: randomly assigned (1/2 each)

Sample size: 2,000 consumers

The Data

```
# A tibble: 6 x 4
     id treatment_status pre_spend post_spend
  <dbl>            <dbl>     <dbl>      <dbl>
1     1                0     133.        97.7
2     2                1     107.        72.5
3     3                0      90.1       88.9
4     4                0      36.4       31.5
5     5                0     151.       162. 
6     6                0      33.6       11.9
```

We also observe consumer behaviour before the test

What We've been Doing So Far

$$spend_i = \beta_0 + \beta_1 Treatment_i + \varepsilon_i$$

```
# A tibble: 2 x 5
  term             estimate std.error statistic p.value
  <chr>               <dbl>     <dbl>     <dbl>   <dbl>
1 (Intercept)         89.7       1.62     55.3   0
2 treatment_status     4.25      2.28      1.86  0.0626
```

What can we improve?

Our **existing estimator is unbiased**

- Which means it delivers the correct estimate, on average.

Potential **improvement**: we could try to **decrease its variance**.

Decreasing the variance of an estimator is important since it allows us to:

- Detect smaller effects
- Detect the same effect, but with a smaller sample size

In general, an estimator with a smaller variance allows us to run tests with a higher power, i.e. ability to detect smaller effects.

CUPED

Suppose you are running an A/B test and Y is the outcome of interest (revenue in our example)

- The binary variable T indicates whether a single individual has been treated or not

Suppose you have access to **another variable** X at the unit level which is **not affected by the treatment**

- And has known expectation $E[X]$.

Can we use X to reduce the variance of the estimate of the average treatment effect?

CUPED

Define:

$$\hat{Y}^{CUPED} = \bar{Y} - \theta\bar{X} + \theta E[X]$$

This is an **unbiased estimator** for E[Y] since last terms cancel out

CUPED

However the **variance of $\hat{Y}^{CUPED}$ is lower** than Y:

$$Var(\hat{Y}^{CUPED}) = Var(\bar{Y})(1 - \rho^2)$$

where ρ is the correlation between Y and X

$\implies$ higher correlation between Y and X $\rightarrow$ higher variance reduction using CUPED

Estimating the ATE with CUPED

$$
\begin{aligned}
\widehat{ATE}^{CUPED} &= \hat{Y}^{CUPED}(T=1) - \hat{Y}^{CUPED}(T=0) \\
&= (\bar{Y} - \theta\bar{X} + \theta E[X] | T=1) - (\bar{Y} - \theta\bar{X} + \theta E[X] | T=0) \\
&= (\bar{Y} - \theta\bar{X} | T=1) - (\bar{Y} - \theta\bar{X} | T=0)
\end{aligned}
$$

Optimal Choice of Pre-Experiment Variable (X)

X should have the following properties:

- **Not affected by the treatment**
- Be as correlated with Y as possible

The authors of the original CUPED paper suggest using **pre-treatment outcome** variables since it gives the most variance reduction in practice.

Computing CUPED Estimate

1. Estimate $\hat{\theta}$ by regressing Y on X
2. Compute $\hat{Y}^{CUPED} = \bar{Y} - \hat{\theta}X$
3. Compute the difference of $\hat{Y}^{CUPED}$ between treatment and control groups

CUPED in Action: Estimating θ

```r
theta <-
    tidy(lm(post_spend ~ pre_spend, data = df)) %>%
    filter(term=="pre_spend") %>%
    select(estimate) %>%
    purrr::pluck('estimate')

print(theta)
```

```
[1] 0.8393084
```

```r
#alternative:
#cov(df$post_spend, df$pre_spend) / var(df$pre_spend)
```

CUPED in Action: Computing $\widehat{Y}_1^{CUPED}$

```
df <-
    df %>%
    mutate(cuped_spend = post_spend -
                theta*(pre_spend - mean(df$pre_spend)
                       )
           )
```

CUPED in Action: Estimate the ATE

```r
mod_cuped <- lm(cuped_spend ~ treatment_status,
                data = df)
tidy(mod_cuped)
```

```
# A tibble: 2 x 5
  term              estimate std.error statistic p.value
  <chr>                <dbl>     <dbl>     <dbl>   <dbl>
1 (Intercept)          89.0       1.24     72.0  0
2 treatment_status      5.55      1.74      3.19 0.00144
```

An Equivalent Formulation

1. Regress Y on X and compute the residuals, $\tilde{Y}$
2. Compute $\hat{Y}^{CUPED} = \tilde{Y} + \bar{Y}$
3. Compute the difference between $\hat{Y}^{CUPED}$ between the treatment and control group

An Equivalent Formulation

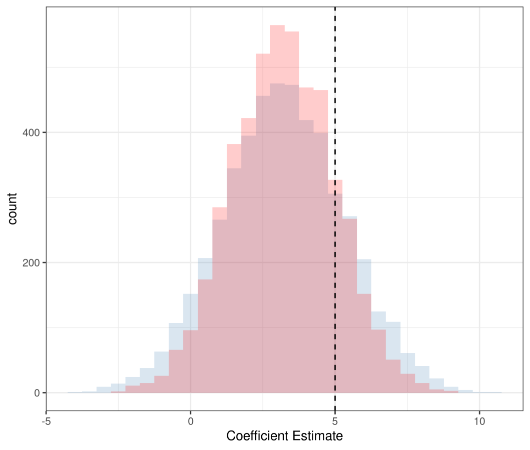
```
df <-
    df %>%
    mutate(post_spend_tilde =
                resid(lm(post_spend ~ pre_spend,
                         data =df
                         )
                      ) +
                mean(df$post_spend)
           )

tidy(lm(post_spend_tilde ~ treatment_status, data = df))
```

```
# A tibble: 2 x 5
  term             estimate std.error statistic p.value
  <chr>               <dbl>     <dbl>     <dbl>   <dbl>
1 (Intercept)         89.0       1.24     72.0  0
2 treatment_status     5.55      1.74      3.19 0.00144
```

CUPED Performance

Comparison of CUPED vs “standard” estimate over 5000 simulated datasets from the same DGP



Summary

- **CUPED aims to decrease the variance of the ATE** by leveraging additional consumer data that is unaffected by the experiment
- **CUPED transforms the outcome variable**, then we use our **conventional toolkit** to analyse the transformed data
- CUPED decreases variance **by using the additional data to make differences between groups "clearer"**

2/ Clustered Standard Errors

A Problem We Need to Solve

Unit of **treatment assignment differs from the unit of observation**

- Example 1: treat all customers in a certain region while observing outcomes at the customer level,
- Example 2: treat all articles of a certain brand, while observing outcomes at the article level.

Usually this happens because of practical constraints with how we can randomize

Implication: Treatment effects are “not independent” across observations

- Example 1: Customer in a region is treated, also other customers in the same region will be treated
- Example 2: If one article of a brand not treated, neither are any of the others

In our **inference** we have to **take this dependence into account**

Example: Customer Order Data and Recommenders Redux

Business questions: Does showing a carousel of related articles at checkout to incentivize customers to add other articles to their basket?

Test setting: Online Website, carousel introduction

Unit: A consumer

Treatments: control group, adding a carousel after adding an item to cart

Reponse: spending in the next 28 days

Selection: all consumers who purchased in last 60 days

Assignment: Display carousel to **consumers** at random

Sample size: 2,000 consumers

Load the Data

```
# A tibble: 6 x 3
   user treatment_status revenue
  <dbl>            <dbl>   <dbl>
1     1                1   192.
2     2                1    91.3
3     3                1    45.6
4     4                1   101.
5     5                0    88.2
6     6                0    15
```

Question: Do we see the same consumers make more than one purchase?

Question: If so, why might this be a problem?

Estimate the ATE

```
# A tibble: 2 x 5
  term             estimate std.error statistic p.value
  <chr>               <dbl>     <dbl>     <dbl>   <dbl>
1 (Intercept)        4.38      0.0224    195.     0
2 treatment_status   0.0642    0.0311      2.07  0.0390
```

Question: What assumptions have we made about the distribution of the error term when we compute the standard error this way?

“Default” Standard Errors

*By default, R assumes **homoskedastic standard errors**:

$$Var(\varepsilon_i|X_i) = \sigma^2$$

and between any two observations:

$$Cov(\varepsilon_i, \varepsilon_j|X_i) = 0$$

In our setting:

- Variance of the error term is the same across consumers
- Covariance of error term is the same across consumers is zero
- **Covariance of error term between multiple purchases of the same consumer is zero**

Relaxing Homoskedasticity: Heteroskedasticity

Let's weaken these assumptions step by step:

- ~~Variance of the error term is the same across consumers~~
- **Variance of the error term is different across consumers**
- Covariance of error term is the same across consumers is zero
- Covariance of error term between multiple purchases of the same consumer is zero

$$Var(\varepsilon_i|X_i) = \sigma_i^2$$

Different assumption on $Var(\varepsilon_i|X_i) \implies$ different formula to compute standard error

- We'll skip the math (hurrah!)

Heteroskedasticity Robust Standard Errors

```
Call:
lm_robust(formula = log(revenue) ~ treatment_status, data = recommender,
    se_type = "HC1")

Standard error type:  HC1

Coefficients:
                 Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper   DF
(Intercept)       4.38103    0.02311 189.604  0.00000 4.335724   4.4263 2548
treatment_status  0.06423    0.03117   2.061  0.03945 0.003107   0.1253 2548

Multiple R-squared:  0.001671 , Adjusted R-squared:  0.00128
F-statistic: 4.246 on 1 and 2548 DF,  p-value: 0.03945
```

Question: Do we see much of a difference in this case?

Relaxing Homoskedasticity: Clustering

- Let's weaken these assumptions step by step:
  - ~~Variance of the error term is the same across consumers~~
  - Variance of the error term is different across consumers
  - Covariance of error term is the same across consumers is zero
  - ~~Covariance of error term between multiple purchases of the same consumer is zero~~
  - **Covariance of error term between multiple purchases of the same consumer is non-zero**

For any two observations of the **same consumer**, g:

$$Cov(\varepsilon_{ig}, \varepsilon_{jg} | X_g) = \rho_g \sigma_{ig} \sigma_{jg}$$

Different assumption $\implies$ different standard error!

Cluster Robust Standard Errors

```
Call:
lm_robust(formula = log(revenue) ~ treatment_status, data = recommender,
    clusters = user)

Standard error type:  CR2

Coefficients:
                 Estimate Std. Error t value Pr(>|t|)  CI Lower CI Upper     DF
(Intercept)       4.38103    0.02546 172.106  0.00000  4.331070   4.4310  844.4
treatment_status  0.06423    0.03470   1.851  0.06434 -0.003829   0.1323 1750.7

Multiple R-squared:  0.001671 , Adjusted R-squared:  0.00128
F-statistic: 3.426 on 1 and 1999 DF,  p-value: 0.06432
```

Question: Is there a difference now?

Summary

- If you **assign treatment at a higher level than your unit of observation**, you **need to correct the standard errors** in your analysis
- You should **cluster your standard errors** at the **level at which the treatment was allocated**
    - In our example: the consumer
- Cluster-robust standard errors are larger than the usual standard errors only if there is dependence across observations.
    - If observations are only mildly correlated across clusters, then cluster-robust standard errors will be similar to homoskedastic ones.

3/ The SUTVA assumption

SUTVA

So far we have made an (implicit) assumption:

"We know what the treatment is"

More precisely:

1. The **potential outcomes** for each unit **do not vary with the treatment assigned to other units** (no interference)
2. For each unit, there are **no different versions** of each **treatment** level (no hidden variation of treatments)

This is known as the **Stable Unit Treatment Value** (SUTVA) assumption

Why Might SUTVA Fail in Online Experiments?

The Stories team at Instagram tries to understand the effect of a new product feature

- e.g., a new emoji reaction

on user engagement, measured by the time on the app.

A simple randomization strategy at the user level assigns half of the population into the treatment group and the other half in the control.

Question: How does SUTVA fail here?

Why Might SUTVA Fail in Online Experiments?

Answer:

Users are connected on the platform, the control group increases (or decreases) the time spent on the app as their treated friends increase (or decreases) the engagement.

Is it a BIG deal?

In short, **yes!**

- Empirical studies and simulations show that the bias from interference ranges from 1/3 to the same size as the treatment effect
- It may mess with the direction of the treatment, e.g., turns positive effect into negative; vice versa.

Solutions to SUTVA Violations

Common solutions used in large (tech) companies

1. Coarser Levels of Randomization
2. Ego-Cluster Randomization
3. Switchback designs

We'll briefly talk about each one

Coarsening the Level of Randomization I

A ride-sharing company (e.g., Lyft) wants to check if a new matching algorithm improves User Retention.

Question: Can we randomly assign riders into the treated/non-treated conditions and compare the group means?

Coarsening the Level of Randomization II

Answer: No, because riders in the same geographic location share the same driver pool and any user change in one group affects the other due to the limited driver pool.

Solution: Administer the randomization process at the coarser level (e.g., city, region).

Coarsening the Level of Randomization III

A tradeoff is present:

- Coarser granularity $\rightarrow$ fewer units
- Larger variance
- Less statistical power

Rule of Thumb: Aggregate data up to the granularity level that each unit won't interact with each other and we will still have a sufficient number of observations.

Real World examples:

- Ridesharing Marketplaces, Lyft
- Netflix

Ego-Cluster Randomization I

LinkedIn wants to know how a new introducing new reactions to posts impact engagement metrics on the platform

Question: Can they run a standard A/B test? Why or why not?

Ego-Cluster Randomization II

Answer: No, there is a high level of interference among connected users.

- Giving the treatment to Person A and all of A's connections will be treated as well,
    - Eegardless of their original treatment status.
- A simple A/B test is not possible.

The **interference** is not caused by geographic locations but by **connected networks**.

Ego-Cluster Randomization III

Solution: Ego-cluster randomization, which treats a focal person ("ego") and her immediate connections ("alter") as a cluster, then randomizes the treatment assignment at the cluster level.

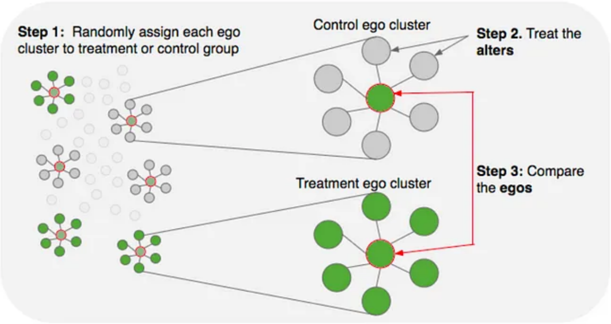


Figure 1: High level diagram of the method

Switchback Designs I

UberEats wants to test out how dynamic pricing (i.e., extra charge for rush hours) would affect customer experience, measured by User Retention.

Question: Why won't A/B tests at the user level work?

Switchback Designs II

Answer: If we randomly assign half of the population into the treatment and the other half to the control group, the treated group only see half of the benefit of supply equilibration,

- The **control group still gets the partial benefit** without the extra cost

… which is a violation of the SUTVA assumption.

Switchback Designs III

Solution: Chooses a higher level of analysis ...

... and randomize the treatment at distinct geography and time window

- Is called a Switchback design.
- The design toggles the treatment on and off at the distinct geography-time level and checks the changes in the outcome variables

Switchback designs IV

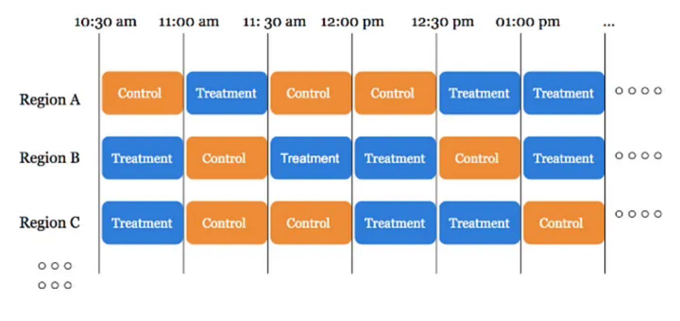


Illustration of time-market unit randomization

Switchback design assumes dependence within the clusters but independence among clusters.

4/ Recap

Summary

- CUPED decreases the variance of A/B test estimates by leveraging pre-treatment data that is unaffected by the experiment
- Robustifying standard errors at the unit of treatment prevents incorrect statistical inference
- Alternative A/B testing designs offer ways around violations of the SUTVA assumption

Acknowledgements

I have borrowed and re-mixed material from the following:

- Matteo Courthoud's "Clustered Standard Errors in A/B Tests" and "Understanding CUPED"
- Chi Huang's "What is SUTVA and What to Do When It's Violated in Practice"

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