Causality & Difference in Differences

Social Media and Web Analytics

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Learning Goals for this Week

- Explain the the difference between correlation and causation
- Understand the difference between regression assumptions and causal assumptions
- Explain the terms Randomized Control Trial and Natural / Quasi Experiment
- Define the term 'Difference in Differences'
- Estimate treatment effects using Difference in Differences
- Reflect on assumptions underlying causal claims from Difference in Difference estimates

Causality

Why Causality?

- Many questions we want answers to are **causal**
- When we talk about marketing, we often want to know why something happens
  - Did demand/revenue/... change because of ?
  - And by how much?
- We also care about non-causal questions (prediction, descriptive evidence)
  - But our comparative advantage should be causality

Why Causality as a Marketing Analyst?

- Causality should be a marketing analyst's **comparative advantage**
  - Plenty of fields do statistics, many probably do it better
  - Few fields worry about causality and the *why* questions the way we (should) do
- We can design more effective marketing strategies if we can identify causal effects
  - Which will generate a boost in KPIs
- **Skill to acquire**: Understanding when to make causal claims and when not
  - Your value to a future employer sky rockets if you can do this well

What is Causality?

X causes Y if …

- We intervene and change X and nothing else
- Then Y changes as a result

Examples of Causal Relationships

Obvious:

- Turning on a light switch causes a light to be on
- Fireworks raise the noise level

Not so obvious:

- TV Advertising increases product demand
- Tweets about movies increase demand for it at theatres

Remark: The **size** these effects are **much smaller** than you probably think

Examples of Non-Causal Relationships

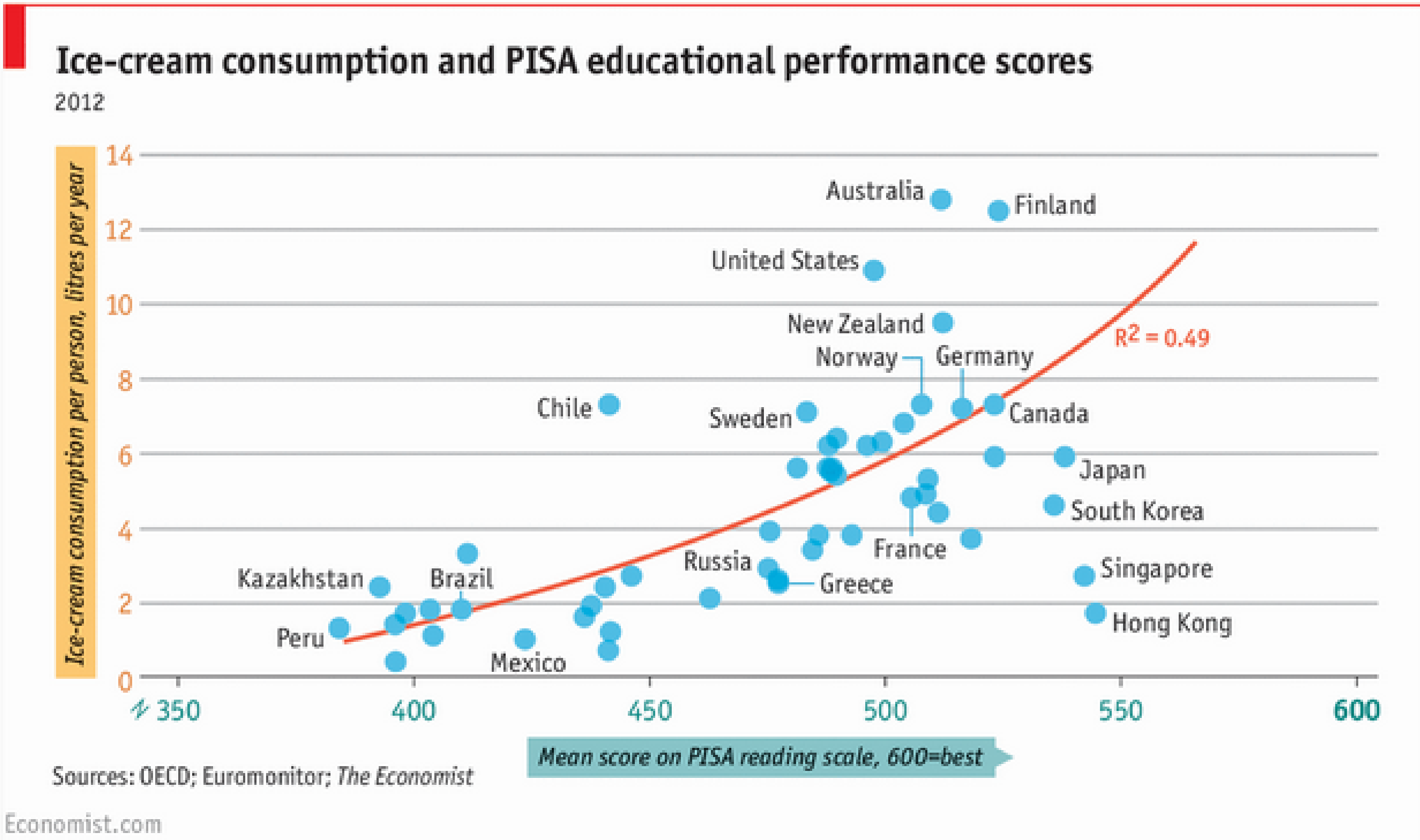
Obvious:

- Number of people wearing shorts at the beach and ice cream consumption
- Roosters crowing followed by sunrise

Some not so obvious:

- School vending machines and obesity
- Search engine advertising and revenue (in the short term!)

Correlation is not Causation



Why Correlation is not Causation

(Some) possible reasons **A** might not cause **B**:

- **The opposite is true**
  - B actually causes A
- The two are correlated, but **there's more to it**:
  - A and B are correlated, but they're actually caused by C
- There's **another variable involved**:
  - A does cause B as long as D happens
- There is a **"chain" reaction**:
  - A causes E, which leads E to cause B
  - ... but you only saw that A causes B from your own eyes
- It's due to **chance**

The Difficulty of Causal Inference

Can we tell when correlation $\implies$ causation?

- Answer 1: It's *hard*
- Answer 2: It is possible, but we *need assumptions*

What kind of assumptions?

- "What would have beens" - i.e. (approximate) counterfactual outcomes
- "As good as random" - i.e. no selection on unobservables
  - Known as "conditional independence"
  - Intuition: Given some control variables, differences in variable we care about are only due to randomness
  - No unobserved factors driving variation in variable of interest

Even then:

- At *best* we'll estimate an **average causal effect**

Regression and Causality

Regression assumptions on their own

$$\neq \quad \textbf{causal interpretations of } \beta$$

- **Regression assumptions**: Unbiasedness, Variance of estimates
- "**Causal Inference assumptions**": Can an unbiased estimate be interpreted causally
  1. Valid counterfactual outcomes
  2. Conditional independence

Note: Cannot test these assumptions 'statistically'

Experiments in Marketing Analytics

Recent trend: use **'experiments' to estimate causal effects**

- Why? Clear counterfactual outcomes, reasonable to assume conditional independence

Experiments in Marketing!?

Yes. Two kinds ...

- **Randomised Control Trial (RCT)**
  - Researcher randomly assigns observational units to treatment group, control group
- **Natural Experiments / Quasi-Experiments**
  - "Nature" divides population into treatment and control in a way that is "as good as random"

Both approaches: Compare changes over time between groups

- How? ... that's what is coming next

Difference in Differences

What is Difference in Differences?

Want to answer the following question:

What is the effect of some marketing intervention on those who were effected by it?

- Call the intervention a **treatment**
- The treatment takes one of two values:
  - treatment = 1 if an observation is effected by the treatment
  - treatment = 0 if an observation is not effected by the treatment
- Observations are **treated at random**
- The treatment effects an **outcome**:

Estimator I: Before vs After?

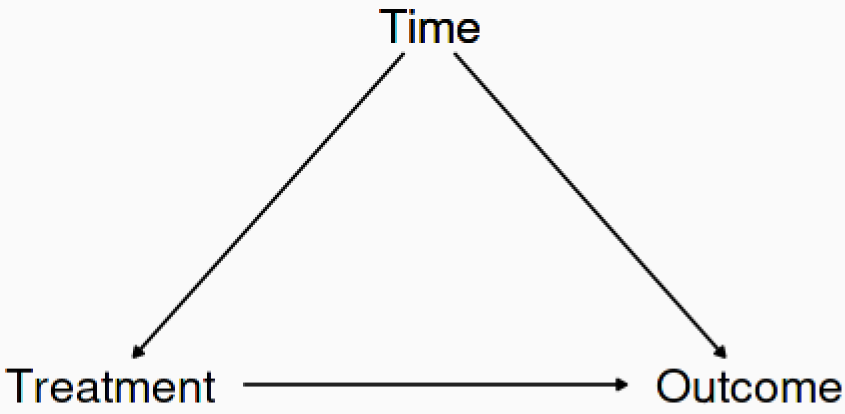
- We have data on observations **before** and **after** a treatment is introduced
- Let $\bar{y}$ denote averages

Proposed estimator I: **Before vs After for Treatment Group**

$$\text{Treatment Effect} = \bar{y}_{\text{after}} - \bar{y}_{\text{before}}$$

This **will not work**. Why?

- Time: things change over time for reasons unrelated to treatment



Estimator I: Before vs After?

Can't we control for time via (say) regression!?

- **No**
  - **treatment** occurrence and **time** are perfectly correlated
- Observation is either:
  - Before and Untreated, or
  - After and Treated.
- If control for time, you're comparing people with the same values of Time ...
- ... who must also have the same values of Treatment!

$\Longrightarrow$ Estimator won't work

Estimator II: Treatment vs Control

- We have data on observations for **treated** and **untreated** after the treatment is introduced
- Let $\bar{y}$ denote averages

Proposed estimator II: **Treated vs Untreated in the After Period**

$$\text{Treatment Effect} = \bar{y}_{\text{treated}} - \bar{y}_{\text{untreated}}$$

This **will not work**. Why?

- Treatment group might naturally vary from control group

$\implies$ Difference between them could be due to:

- The intervention, or
- Uncontrolled differences between the two groups

$\implies$ Estimator won't work

Difference in Differences

- Previous estimators: one difference (one minus sign)
    - **They don't work**

Why?

- Estimator I: confounded by time differences
- Estimator II: confounded by group differences

What if we could combine ideas from both?

$\implies$ that is what difference in differences does

Cool! How?

Difference in Differences: Notation

Assumption: The effect of time is constant between treated and control groups

We need four averages:

1. Control group, before intervention starts

$$\bar{y}^{\text{control}}_{\text{before}} = \beta_0$$

2. Control group, after intervention starts

$$\bar{y}^{\text{control}}_{\text{after}} = \beta_0 + \beta_1$$

3. Treatment group, before intervention starts

$$\bar{y}^{\text{treatment}}_{\text{before}} = \beta_0 + \beta_2$$

4. Treatment group, after intervention starts

$$\bar{y}^{\text{treatment}}_{\text{after}} = \beta_0 + \beta_2 + \beta_1 + \delta$$

$\Longrightarrow$ the (average) treatment effect is δ

This looks easier in a table...

The Difference in Difference Table

	Before	**After**
Control	β_0	$\beta_0 + \beta_1$
Treatment	$\beta_0 + \beta_2$	$\beta_0 + \beta_2 + \beta_1 + \delta$

The Difference in Difference Table

	Before	After	After - Before
Control	β_0	$\beta_0 + \beta_1$	β_1
Treatment	$\beta_0 + \beta_2$	$\beta_0 + \beta_2 + \beta_1 + \delta$	$\beta_1 + \delta$
Treatment - Control			δ

'Double Differencing' $\implies$ estimate δ

I call this DiD estimate using averages **simple DiD**

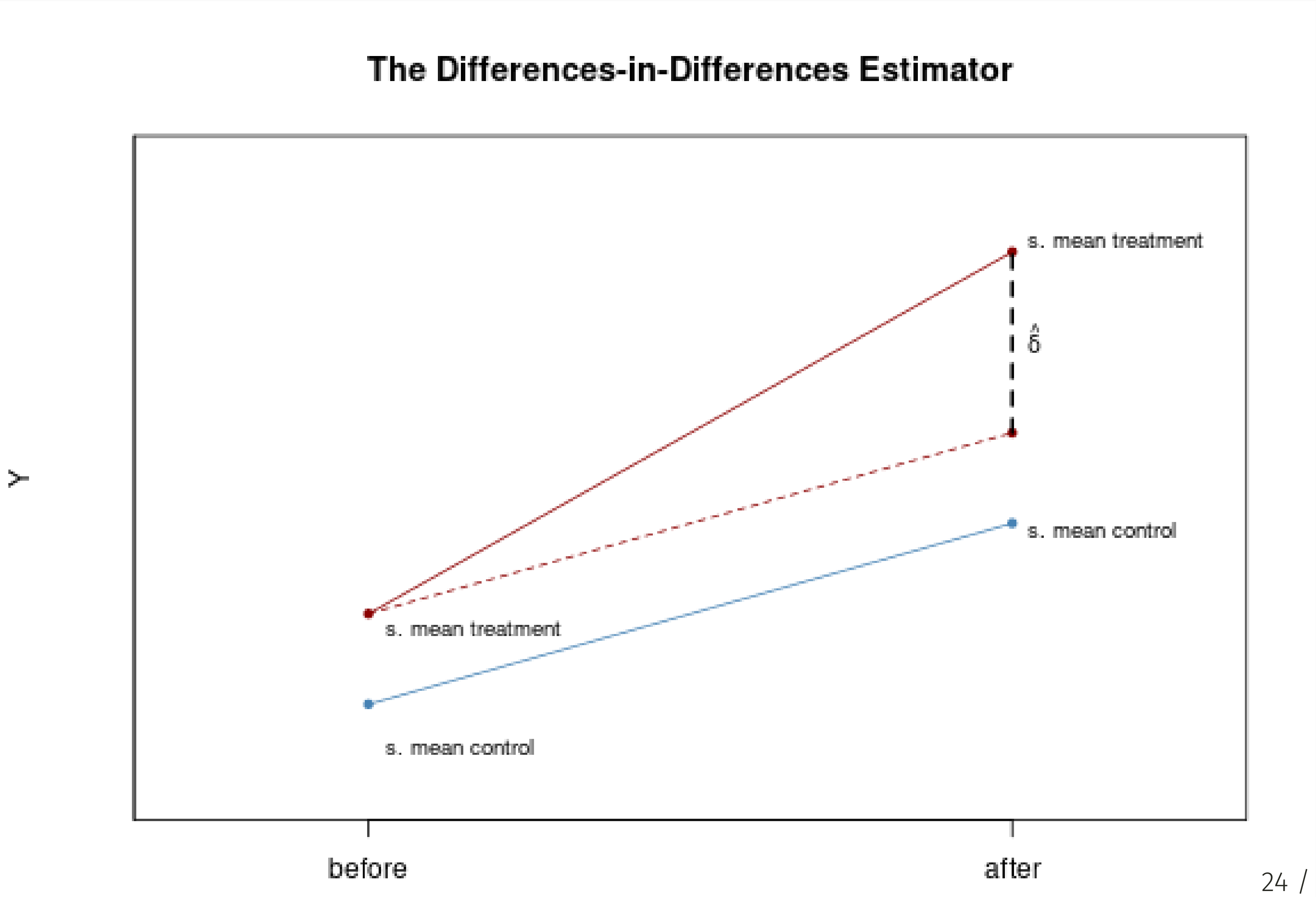
The Difference in Difference Table

	Before	After	After - Before
Control	β_0	$\beta_0 + \beta_1$	
Treatment	$\beta_0 + \beta_2$	$\beta_0 + \beta_2 + \beta_1 + \delta$	
Treatment - Control	β_2	$\beta_2 + \delta$	δ

'Double Differencing' $\implies$ estimate δ

I call this DiD estimate using averages **simple DiD**

Difference in Difference Graphically



Difference in Difference in R

How can we do this in R?

Let's first create some data:

- years: 2002 - 2010
- treatment for some observations in year 2007
- treatment effect: 2

```
# Create our data
diddata <- tibble(year = sample(2002:2010,10000,replace=T),
                  group = sample(c('TreatedGroup','UntreatedGroup'),10000,replace=
  mutate(after = (year >= 2007)) %>%
  #Only let the treatment (i.e. Treatment) be applied to the treated group
  mutate(Treatment = after*(group=='TreatedGroup')) %>%
  mutate(Y = 2*Treatment + .5*year + rnorm(10000)) %>%
  select(-Treatment) %>%
  mutate(treatment = case_when(
    group == "TreatedGroup" ~ TRUE,
    TRUE ~ FALSE
    )
  )
```

Difference in Difference in R

Now, compute averages by group and treatment status

```
means ←
  diddata %>%
  group_by(group,after) %>%
  summarize(Y=mean(Y)) %>%
  ungroup()

print(means)
```

```
## # A tibble: 4 × 3
##   group          after      Y
##   <chr>          <lgl> <dbl>
## 1 TreatedGroup   FALSE 1002.
## 2 TreatedGroup   TRUE  1006.
## 3 UntreatedGroup FALSE 1002.
## 4 UntreatedGroup TRUE  1004.
```

Difference in Difference in R

As a 'table'

```
did_table <-
  means %>%
  pivot_wider(names_from = after,
              values_from = Y
              )
print(did_table)
```

```
## # A tibble: 2 × 3
##   group          `FALSE` `TRUE`
##   <chr>            <dbl>  <dbl>
## 1 TreatedGroup     1002.  1006.
## 2 UntreatedGroup   1002.  1004.
```

Difference in Difference in R

Compute Treatment Effect, $\hat{\delta}$

```r
#Before-after difference for untreated, has time effect only
bef_aft_untreated <- filter(means,group=='UntreatedGroup',after==1)$Y -
                     filter(means,group=='UntreatedGroup',after==0)$Y
#Before-after for treated, has time and treatment effect
bef_aft_treated <- filter(means,group=='TreatedGroup',after==1)$Y -
                   filter(means,group=='TreatedGroup',after==0)$Y
#Difference-in-Difference! Take the Time + Treatment effect,
#                          and remove the Time effect
did <- bef_aft_treated - bef_aft_untreated

print(paste("Diff in Diff Estimate: ", did))
```

```
## [1] "Diff in Diff Estimate:  1.97404126317736"
```

Is Our Estimate Causal

We need **two assumptions** for causality:

1. A **valid counterfactual outcome** to compare treated group to
    - The control group gives us this
2. **Conditional Independence**: treatment assignment "as good as random"
    - We randomly assigned the treatment to some observations

$\implies$ **Difference in difference can give is causal estimates of the average treatment effect!**

Difference in Differences as a Regression

DiD as a Regression

$$y_{it} = \beta_0 + \beta_1 After_t + \beta_2 Treated_i + \delta After_t \times Treated_i + \varepsilon_{it}$$

where:

- $After_t$ = 1 in the period after treatment occurs, zero otherwise
- $Treated_i$ = 1 if the individual is ever treated, zero otherwise

DiD as a Regression

$$y_{it} = \beta_0 + \beta_1 After_t + \beta_2 Treated_i + \delta After_t \times Treated_i + \varepsilon_{it}$$

- β_0 is the prediction when $Treated_i = 0$ and $After_t = 0$
  - $\rightarrow$ the Untreated Before mean!
- β_1 is the *difference between* Before and After for $Treated_i = 0$
  - $\rightarrow$ Untreated (After - Before)
- β_2 is the *difference between* Treated and Untreated for $After_t = 0$
  - $\rightarrow$ Before (Treated - Untreated)
- δ is *how much bigger the Before-After difference* is for $Treated_i = 1$ than for $Treated_i = 0$
  - $\rightarrow$ (Treated After - Before) - (Untreated After - Before) = Treatment Effect!

Let's see that in action with `R`

DiD as a Regression

```r
reg_did <- lm(Y ~ after*treatment, data = diddata)

tidy(reg_did, conf.int = TRUE)
```

```
## # A tibble: 4 × 7
##   term                    estimate std.error statistic p.value conf.low conf.high
##   <chr>                      <dbl>     <dbl>     <dbl>   <dbl>    <dbl>     <dbl>
## 1 (Intercept)              1.00e+3    0.0226 44409.      0      1.00e+3 1002.
## 2 afterTRUE                2.29e+0    0.0341    67.1     0      2.22e+0    2.35
## 3 treatmentTRUE           -9.34e-3    0.0320    -0.292   0.770 -7.21e-2    0.0534
## 4 afterTRUE:treatmentTR…   1.97e+0    0.0484    40.8     0      1.88e+0    2.07
```

Advantages of Regression Approach

1. **Get standard error of the estimate**
    - Assess whether effect is statistically significant
    - *Should cluster standard errors*
    - (see this week's reading for suggestions on how)
2. **Can add extra control variables into the regression**
    - Either as 'usual' controls and/or as fixed effects
    - Particularly useful for Natural / Quasi Experiments
    - (see this week's reading)
3. **Can use $log(y)$ as dependent variable**
    - $\rightarrow \hat{\delta}$ is the percentage change in y due to the treatment

Hidden Assumptions, Caveats, etc

Hidden-ish Assumption: Parallel Trends

I briefly mentioned this in passing...

We must assume that Time effects treatment and control groups equally

- Otherwise controlling for time (i.e. `after`) won't work

This is called the **parallel trends** assumption

- Again, *if the Treatment hadn't happened to anyone*, the differences between the treatment and control would stay the same

Checking for Parallel Trends

Like many assumptions - its **untestable**

- Though we can **'check' whether patterns in the data are suggestive its OK**
- Here's one way:
  - Are *prior trends* are the same for Treated and Control groups
  - Generally, compute average of outcome by group over time
  - (needs multiple pre-treatment periods)
  - Was the gap changing a lot during that period? If not, suggestive we're OK

"As good as random" Redux

Remember our two assumptions for causality:

1. **Valid counterfactual outcomes**
    - Control Group solves this one for us
2. **Conditional independence**: nothing unobserved is causing selection into treament group
    - Trickier ...
    - Randomised Control Trial $\longrightarrow$ You're more than likely gonna be OK
    - Natural / Quasi Experiment - have you got a credible proxy for random assignment?
    - Profession's thoughts: Large, visible, unexpected shocks

Threats to Validity

Internal Validity: statistical inference made about causal effects are valid for the considered population

External Validity: inferences and conclusion are valid for the study's population and can be generalized to other populations and settings

Threats to Internal Validity

- **Failure to Randomise**
- **Failure to Follow Treatment Protocol**
- **Attrition**
- **Experimenter Demand Effects**
- **Small Sample Sizes**

Threats to External Validity

- **Non-representative sample**
- **Non-representative Marketing Intervention / Policy**
- **General Equilibrium Effects**

A Warning!

- DiD's popularity is relatively recent, so we're still learning a lot about it!
  - Most relevant has to do with **staggered roll out DiD**
- The regression version of DiD doesn't *necessarily* need to have treatment applied at *one* particular time
  - Treatment could be gradually implemented over time
- Nothing we've explicitly said would prevent us from using the regression DiD right!?
  - Well... that's what we thought for a long time.
  - And you'll see many of published studies doing this.
  - BUT it turns out to actually **bias results by quite a lot**
- There are more complex, newer estimators for staggered roll out case,
  - Too much for this class

Recap

Recap

- Many marketing questions require causal answers
- Establishing causality is goes beyond finding (partial) correlations in data
- RCT and Natural/Quasi Experiments introduce "as good as random" allocation to a treatment / marketing intervention
- Can use Difference in Difference to estimate causal effects of above experiments

Acknowledgements

Material in this set of slides borrows from the great work of others:

- Nick C Huntington Klein's course on Causality and Analytics
- Ed Rubin's Econometrics III
- Alan Spearot's class notes from Econ 113 in Fall 2014
- Hanck et al's Econometrics with R
- Goldfarb & Tucker's Conducting Research with Quasi-Experiments: A Guide for Marketers

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```

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